The LogNormal Mixture Variance Model & Its Applications in Derivatives Pricing and Risk Management

Summary
The LNVM model is a mixture of lognormal models and the model density is a linear combination of the underlying densities, for instance, log-normal densities. The resulting density of this mixture is no longer log-normal and the model can thereby better fit skew and smile observed in the market. The model is becoming increasingly widely used for interest rate/commodity hybrids.

In this first part of my review of the model, I examine the mathematical framework of the model in order to gain an understanding of its key features and characteristics. In a later second stage of the review we will extend our analysis of the mathematical framework and assess the implementation of the model and its application to price and hedge more complex derivative structures.

Model Description

Let’s assume that the forward rate $F$ follows a log-normal process under the forward measure $Q$ as follows:

$$dF(t) = V(t,F)F(t)dW$$

As it’s well known that in the Black-Scholes framework the model does not show any skew or smile. One way to overcome this is to combine several same distributions, for instance, log-normal distributions, but with different means and variances. Let’s define $N$ underlying processes as below:

$$dG_i(t) = \mu_i(t)G_i(t)dt + \sigma_i(t)G_i(t)dW, \quad G_i(0) = F(0), i = 0, 1, \ldots, N$$

Let $\lambda_i$ be the probability of the forward process $F$ following the $i$th underlying dynamics and $\sum_{i=1}^{N} \lambda_i = 1$.

Let $p(t,x)$ be the density for the forward rate process and $p_i(t,x)$ be the density function for the $i$th underlying process so that $p(t,x) = \sum_{i=1}^{N} \lambda_i p_i(t,x)$. In order to keep the mean of the forward rate unchanged, the following condition should be satisfied:

$$\sum_{i=1}^{N} \lambda_i e^{\int_{0}^{t} \mu_i(s)ds} = 1$$

We can verify that the mean is actually kept unchanged:

$$\int_{0}^{\infty} Fp(t,F)dF = \sum_{i=1}^{N} \lambda_i \int_{0}^{\infty} G_i p(t,G_i)dG_i = \sum_{i=1}^{N} \lambda_i G_i(0)e^{\int_{0}^{t} \mu_i(s)ds} = F(0) \sum_{i=1}^{N} \lambda_i e^{\int_{0}^{t} \mu_i(s)ds} = F(0)$$

As to the form of the volatility term in $F$, the process must solve Kolmogorov forward equation (Fokker–Planck equation). We have:

$$\frac{\partial}{\partial t} p(t,x) = \frac{1}{2} \frac{\partial^2}{\partial x^2} \left[ V^2(t,x) x^2 p(t,x) \right]$$

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1 Although we use forward rate as an example, this equally applies to other asset classes, such as FX, equity, etc.
The underlying processes \( G_i \) also must solve Kolmogorov forward equation:

\[
\frac{\partial}{\partial t} p_i(t, x) = -\frac{\partial}{\partial x} \left[ \mu_i(t) x p_i(t, x) \right] + \frac{1}{2} \frac{\partial^2}{\partial x^2} \left[ \sigma_i^2(t, x) x^2 p_i(t, x) \right]
\]

By the linearity of the derivative operator, we have:

\[
\sum_{i=1}^{N} \lambda_i \frac{\partial}{\partial t} p_i(t, x) = \frac{1}{2} \sum_{i=1}^{N} \lambda_i \left[ \frac{\partial^2}{\partial x^2} \left( V_i^2(t, x) x^2 p_i(t, x) \right) \right]
\]

and

\[
\sum_{i=1}^{N} \lambda_i \frac{\partial}{\partial t} p_i(t, x) = -\sum_{i=1}^{N} \lambda_i \mu_i(t) \frac{\partial}{\partial x} \left[ x p_i(t, x) \right] + \frac{1}{2} \sum_{i=1}^{N} \lambda_i \left[ \frac{\partial^2}{\partial x^2} \left( \sigma_i^2(t, x) x^2 p_i(t, x) \right) \right]
\]

After some simple algebra, we finally obtain:

\[
V_i^2(t, x) = \frac{x^2 \sum_{i=1}^{N} \lambda_i \sigma_i^2(t, x) p_i(t, x) + 2 \sum_{i=1}^{N} \lambda_i \mu_i(t) \int_{x}^{+\infty} y p_i(t, y) dy}{x^2 \sum_{i=1}^{N} \lambda_i p_i(t, x)}
\]

Where

\[
V|t, x| = \sqrt{n \sum_{i=1}^{n} \left( \frac{1}{\sqrt{i \times \frac{t}{2}}} e \frac{1}{\sqrt{i \times \frac{t}{2}}} \left| \frac{\text{Log} \left( \frac{x}{S} \right) \cdot \frac{t}{2} \frac{i}{2} \times \frac{t}{2} \right| \right)}
\]

The above formula is used to calculate the local volatility for F based on the underlying processes of \( G_i \).
Fig 1. Local Volatility Surface for Lognormal Mixture Model with $S_0=100; \eta=(0.5,0.1,0.2); \lambda=(0.2,0.3,0.5); r=0.035; q=0.035$

Brigo and Mercurio show that under standard assumptions of continuity in the $\sigma$, the SDE has a unique solution whose marginal density is given by the mixture of lognormals.

$$p(t, x) = \prod_{i=1}^{n} \frac{1}{x \sqrt{2\pi i t}} \exp \left[ -\frac{1}{2} \left( \frac{\ln \left( \frac{x}{S_0} \right) - r \left( \frac{1}{2} \frac{1}{i t} \right)}{\sqrt{\frac{\lambda}{i \theta}}} \right)^2 \right]$$
To calculate the undiscounted value of a call option with strike $K$, we have:

$$
call = E^{Q}[(F - K)^+] = E^{Q}\left[\sum_{i=1}^{N} \lambda_i (G_i - K)^+\right] = \sum_{i=1}^{N} \lambda_i E^{Q}[(G_i - K)^+]$$

We can see that the option value is actually the weighted average of option values from the underlying processes, given explicitly as:

$$
P = \prod_{i=1}^{n} \left| \frac{\text{Ln} \frac{S}{k} + t \omega \frac{1}{2} }{i \sqrt{t}} \right| \left( \frac{\text{Ln} \frac{S}{K} + t \omega \frac{1}{2} }{i \sqrt{t}} \right)^{-k}$$

Where $\Phi$ is the cumulative Normal distribution function, $\omega$ is -1 for a put and +1 for a call and

$$
\prod_{i=1}^{n} \sqrt{\frac{1}{a_i^2 + \frac{1}{2} - u_i^2}}
$$
The option price leads to a wide variety of shapes in the implied volatility structure, such as the smile illustrated in figure 3 below. As demonstrated in figure 4, smiles produced by the LNVM model tend to become increasingly pronounced as time to option expiration decreases, in line with observed behavior in the great majority of derivatives markets.

Note, however, that smiles are symmetric and in order to produce volatility skews, we need to extend the model to allow the underlying stochastic process to be shifted.

Fig 3. Implied Volatility Curve for Lognormal Mixture Model with
$S_0=100; \eta=(0.5, 0.1, 0.2); \lambda=(0.2, 0.3, 0.5); T=1; r=0.035; q=0.035$
A more general model can be created by shifting the original process with an affine transformation of the form:

$$A_t = A_0 + \alpha t, \quad S_t$$

Where $\alpha$ is a real constant.

By Itô’s lemma, the asset price process evolves according to:

$$dA_t : \frac{dA_t}{A_t} = dt + A_t \cdot \frac{dA_0}{A_0} + A_t \cdot \frac{dW_t}{A_t}$$

If $k \cdot A_0 + \alpha t > 0$, we can write the option price as follows:

$$P_i : P_i \left( \sum_{k=1}^{n} \left( \frac{\ln \left( \frac{1}{k} + \frac{A_0}{A_0 + \alpha t} \right)}{i \sqrt{t}} + \frac{1}{2} t i^2 \right) \right)$$

For $\alpha = 0$ the process $A_t$ naturally coincides with the process for $S_t$, while preserving the drift.
The parameter $\alpha$ affects the shape of the implied volatility surface in two ways. Firstly, changing $\alpha$ produces almost a parallel shift of the surface downwards (upwards) as $\alpha$ is increased (reduced) in value. Secondly, it moves the location of the volatility minimum to the left, (lower strike), when $\alpha > 0$, and to the right, (higher strike) when $\alpha < 0$. Examples of these effects are shown the figures 5 and 6 following.

Fig 5: Implied Volatility Skews for Lognormal Mixture Model with $S_0=100; \eta=(0.5,0.1,0.2); \lambda=(0.2,0.3,0.5); T=0.5 ; r=0.035 ; q=0.035$ and $\alpha = -0.5$ (blue), $\alpha = -0$ (burgundy) and $\alpha = 0.5$ (yellow)
Model Application and Calibration

If the asset is a stock or index paying a continuous dividend yield $q$, we assume that interest rates are constant and equal to $r$ for all maturities. Then every forward measure coincides with the risk-neutral measure having $B(t) = e^{rt}$ as numeraire and the stock price dynamics are as described for the process $A_t$, with $\mu = r - q$. Discrete dividends can be handled by setting $q = 0$ and reducing $A_0$ by the present value of all future dividends. Skews found in equity markets can be fitted satisfactorily with large negative values of $\alpha$, although the quality of the fit is likely to deteriorate for near-term options where the skews become more pronounced.

For FX assets we again assume that a constant domestic risk-free rate applies, together with a constant foreign risk-free rate $q$. Since FX implied volatility curves tend to be smiles rather than skews, the simpler unshifted version of the model will usually suffice.

For forward Libor rates we again set $r = q$, since the forward rate $F(t, S, T)$ is a martingale under the forward measure $Q_T$. The $\alpha$ parameter again can be expected to play an important role in the calibration of the often pronounced skews in interest rate markets.

Calibration of the LNVM model is rendered slow and somewhat problematic by the large number of degrees of freedom. Brigo and Mercurio recommend using a global search algorithm with a few model parameters and then refining the search with a local algorithm around the last solution found.

The set of constraints $|i|T_j - t| \sqrt{T_i/ T_j} \leq |i|T_j - t| \sqrt{T_i/ T_j}$ must be introduced in order to avoid imaginary values of the $\sigma_i$’s and there is another constraint on the parameter $\alpha$, which must satisfy $k \cdot A_0 \leq A_t$.

Brigo and Mercurio advise that it usually sufficient to limit the number of mixed distributions to 3 or less, and counsel against assuming constancy of volatility over time, since the fit will worsen considerably.

By way of simple illustration, we fit a mixture of two lognormal densities, calibrated to 2-year Libor caplet volatilities with the underlying Libor rate resetting at 1.5 years. The underlying forward rate is 5.32%, the strikes and 4%, 4.25%, 4.50%, 4.75%, 5.00%, 5.25%, 5.50%, 5.75%, 6.00%, 6.25% and 6.50%, with associated mid-volatilities of 15.22%, 15.14%, 15.10%, 15.08%, 15.09%, 15.12%, 15.17%, 15.28%, 15.40%, 15.52% and 15.69%. With $N=2$, $v_i = \eta_i(1.5)$, $i = 1, 2$ and $\lambda_2 = 1 - \lambda_1$, we look for admissible values of $\lambda_1$, $v_1$, $v_2$, and $\alpha$ minimizing the squared percentage differences between model and mid-market prices. The calibrated model parameters are found to be $\lambda_1 = 0.2859$, $\lambda_2 = 0.7140$, $v_1 = 13.02\%$, $v_2 = 19.85\%$, and $\alpha = 0.1538$. The resulting implied volatilities are plotted in figure X, where they are compared with market mid-volatilities.
Model Risk Characteristics

In the following analysis we focus on comparing the risk characteristics of call options with varying strikes and maturities of between 1 month and 5 years, under the standard Black-Scholes model and a mixture of two shifted lognormal distributions with $S_0=100$; $\eta=(0.35, 0.1)$; $\lambda=(0.6, 0.4)$; $r=5\%$; $q=0\%$ and shift parameter $\alpha$ in the range $(-0.5, 0.5)$. We consider the form and properties of various risk sensitivities, being first and second derivatives of the LNVM pricing function with respect to the underlying, $S$, and time to maturity, $t$, for this double-lognormal mixture density. Derivations of the Greeks are given in Appendix 1.

SMILE CHARACTERISTICS

LNVM is a sticky-delta model: the implied volatility smile moves with the underlying. This means that if spot changes, the implied volatility of an option with a given moneyness doesn't change. So if spot moves from $100$ to $110$, LNVM would predict that the implied volatility of the $110$ strike option would be whatever the $100$ strike option's implied volatility was before the move (as these are both ATM at the time). We illustrate this characteristic of the LNVM model in figure 8 below.

Sticky-delta models, such as LNVM, assume that ATM volatility is the rational estimate of the future cost of replicating future options issued now and that, on average, over the long run, ATM volatility should be independent of the level of the underlying. One implication of this is option deltas under LNVM will exceed Black-Scholes deltas, as the analysis in the next section of the report confirms.
Fig. 8: LNVM Implied volatility curves for $S_0 = 90$ (blue), $S_0 = 100$ (burgundy), $S_0 = 110$ (yellow)
SMILE DYNAMICS

The introduction of the shift parameter $\alpha$ in the extended LNVM model enables asymmetric smiles to be calibrated, as required in equity and interest rate markets. A further attractive feature of the LNVM model (both symmetric and extended) is that smile dynamics parallel the characteristic behavior of many markets, in which the smile becomes increasingly pronounced nearer to expiration. Both features are illustrated in the example in figure 9. However, although the smile dynamics in LNVM are broadly in line with market behavior, the model is likely to perform less well for near-term maturities, where smiles are often much more pronounced and where smile dynamics are often more successfully captured by some form of stochastic jump model. This concern applies more to equity rather than FX markets, in which “U” shaped smiles are roughly similar for all maturities.

Fig 9. Implied Volatility Surface for Lognormal Mixture Model with $S_0=100; \eta=\{0.5,0.1,0.2\}; \lambda=\{0.2,0.3,0.5\}; T<0.2; r=0.035; q=0.035; \alpha=-0.5$

DELTA
The charts in figure 10 compare the behavior of the option delta as the underlying asset changes in value for the Black Scholes model and a lognormal mixture model with shift parameter $\alpha = 0$. As foreshadowed in the discussion of smile dynamics, for the LNVM model option deltas are significantly higher than Black-Scholes for all but the shortest option maturities. In addition, the value of delta as a function of option maturity varies with the shift parameter $\alpha$, as the chart in figure 11 makes clear:

- **For the LNVM model with non-negative $\alpha$, the delta-decay curves** (i.e. the rate of change of option delta over time, approximate that seen in the Black-Scholes model (i.e. the curves are approximately parallel)
- **For the LNVM model with negative $\alpha$, the rate of delta-bleed exceeds that derived from the Black-Scholes model.**
Fig. 10: Comparison of Black-Scholes (upper) and LNVM (lower) option Delta with shift parameter $\alpha=0$. 
Fig. 11: Comparison of Black-Scholes and LNVM ATM option Delta with shift parameter $\alpha = \{-0.5, 0, 0.5\}$

GAMMA

The charts in figure 12 and 13 compare the gamma surfaces for call options with varying strikes and maturities. The well-known characteristics of the surface in the Black-Scholes framework are that gammas decline very quickly for OTM options and accelerate very rapidly for ATM options as they near maturity. The picture for the LNVM model is similar, but with higher gammas for near-the-money options than Black-Scholes.

Of course, the shape of the gamma curve is impacted directly by the shift parameter $\alpha$, increasing in kurtosis with $\alpha$, as demonstrated in figure 14. For negative values of $\alpha$, the LNVM gamma curve has a higher dispersion than the Black-Scholes curve, producing larger option gammas for strikes around $\{-20\%, +40\%\}$ out-of-the-money and lower gammas compared to Black-Scholes around the ATM strikes. As $\alpha$ increases, the position is reversed and LNVM model gamma of ATM options exceeds that of Black-Scholes, while Black-Scholes gammas dominate for OTM options.

In figure 15, we compare gamma as a function of maturity for an ATM option under the Black-Scholes and LNVM models, with different values of the shift parameter $\alpha$. Each of the gamma curves shows its characteristic exponential increase as time to maturity decreases. For options modeled under LNVM with negative shift parameter, option gamma is uniformly lower than that derived under the Black-Scholes framework at all maturities. As the shift parameter is increased, the gamma curve is shifted upwards and for positive $\alpha$ option gammas from the LNVM model exceed those from Black-Scholes at all maturities. So, too, does the rate of change of gamma with respect to time to expiration, $d\Gamma/dt$, increase in absolute magnitude with the shift parameter $\alpha$. 
Fig. X: Black-Scholes option Gamma with shift parameter $\alpha = 0$.

Fig. 13: LNVM option Gamma with shift parameter $\alpha = 0$. 
Fig. 14: Comparison of Black-Scholes and LNVM Gamma for call option with 6m maturity for different values of shift parameter $\alpha$.

Fig. 15: Comparison of Black-Scholes and LNVM Gamma for ATM call option for different values of shift parameter $\alpha$.

**THETA**

Theta, the rate of option decay, is highly non-linear around the ATM strike in the Black-Scholes framework, as shown in figure 16, and the pattern of the theta surface under the LNVM model is much the same (figure 17).
Fig 16: Option Theta from the Black-Scholes model

Fig 17: Option Theta from the LNVM model
The rate of option decay is correlated with the shift parameter $\alpha$, as can be seen from figure 18: the rate of decline in option value increases as $\alpha$ becomes more negative. As in the Black-Scholes model, the rate of option decay accelerates as options move towards expiration and the rate of decay is correlated with the shift parameter $\alpha$, as illustrated in figure 19. Decay rates in the LNVM framework will exceed those in the Black-Scholes model for sufficiently negative values of $\alpha$.

**Fig 18:** ATM option value decay in the Black-Scholes and LNVM model frameworks

**Fig 19:** ATM comparison of Theta in the Black-Scholes and LNVM model frameworks
VOLATILITY RISK

The standard volatility risk sensitivity parameter Vega is problematic to evaluate in the context of a model such as LNVM, which makes use of a volatility term structure rather than a single volatility parameter as in Black Scholes. Instead we focus on a DV01, i.e. the change in option value of a 1% increase in implied volatility.

Fig 20: Volatility sensitivity (DV01) for %1 increase in ATM implied volatility

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In the LNVM context there are a wide variety of ways in which the volatility term structure can be perturbed and we consider both parallel and non-parallel moves. As shown in figure 20, the impact of changes in volatility depends not only on the level and steepness of the volatility curve, but also on the shape parameter $\alpha$. Broadly, as might be expected, changes in option value are inversely related to the shift parameter, largely because of its impact on the level and slope of the volatility skew:

- For $\alpha = 0$, the impact of a 1% increase in volatility on ATM option values is approximately the same under the Black-Scholes and LNVM models, while the relative impact on OTM option values depends critically on whether the change in the volatility curve is assumed to be parallel or non-parallel.

- For $\alpha < 0$, the impact of a 1% increase in volatility is greater under LNVM than Black-Scholes across all strikes (except higher strikes in the vol-flattening scenario).

- For $\alpha > 0$, the impact of a 1% increase in volatility is greater under Black-Scholes than LNVM across all strikes.

The inverse relationship between shift parameter and volatility sensitivity is further illustrated in figure 21.

As in the Black-Scholes framework, in the LNVM model volatility sensitivity is much greater for long-dated options than options close to expiration as illustrated in figure 22.

Fig 21: Impact of shift parameter $\alpha$ on volatility sensitivity (DV01)
Fig 22: Volatility sensitivity (DV01) in the LNVM model as a function of option maturity
Implications for Trading, Hedging and Risk Management

Through judicious selection of an appropriate value of the shift parameter $\alpha$, the extended version of LNVM allows us to calibrate smiles and skews typical of many different markets. For example, a value of $\alpha$ close to zero would be appropriate for modeling the symmetric smiles found in FX markets, while a negative value of $\alpha$ would be used to reproduce the negative skews typically seen in equity markets.

In terms of smile dynamics, however, the LNVM model has important characteristics that may be difficult to reconcile with observed market behaviors. Firstly, it is going to be challenging for a floating (sticky-delta) smile model such as LNVM to reproduce the variable smile-dynamical behavior which characterizes equity markets. These have a persistent negative skew and, during periods of positive returns, typically behave in a way best described by a sticky-strike model: implied volatility tends to fall as the underlying continues to appreciate; ATM options are the most liquid; and these most liquid options are sold more and more cheaply as if traders never had to worry about future market declines. In this regime of market complacency or “greed”, option deltas are close to Black-Scholes levels. Conversely, during periods of stress, market behavior can often best be described by a sticky-IMPLIED-tree model. In this view, the skew is attributable to an expectation of higher volatility as the market declines. ATM volatility falls much more quickly than the skew would appear to imply and option deltas are lower than Black-Scholes deltas. During period of normalcy, we can expect the sticky-delta type of model to describe the skew dynamics quite well and indeed, an argument can be made that, at least in the long term, the regimes of “fear” and “greed” tend to average out to produce the kind of behavior well described by floating smile models like LNVM. However, during periods of rapid appreciation LNVM deltas may significantly over-estimate the sticky-strike deltas pertaining in the market and this delta-overestimation is likely to get even worse in periods of market stress. We have also noted elsewhere in this report that the practitioner is likely to experience difficulty in calibrating the very much steeper smiles for short-dated maturities, which are often more accurately represented as the product of a jump-diffusion process.

Similar concerns may also hold for commodity derivatives or hybrids, given the extended trends in metals, grains, energy and other commodities, and the recent sell-off and increased volatility in those markets. Here, too, we might expect to see changes in smile dynamics, from stick-strike to sticky implied-tree, creating difficulties for the LNVM model in terms of calibration and risk sensitivity estimation.

As far as interest rate markets are concerned, not only are skews typically very sticky, but there is the added complication that they are likely to be calibrated with positive shift parameter under LNVM. As our previous analysis has shown, this will inflate not only deltas, but also gammas, relative to those derived from Black-Scholes, to an even greater extent than in markets where the LNVM model calibrates with a zero or negative shift parameter value.

LNVM is likely to be more successful in modeling FX/hybrid exotics, given the floating smile dynamics that typically characterize currency markets, which are fairly well described by sticky delta models, and where smiles are relative consistent across maturities.
A major concern for traders and risk managers is the complexity of volatility risk in the LNVM model. Volatility sensitivity becomes an increasingly important factor for longer maturities in both the LNVM and Black-Scholes frameworks. However, volatility sensitivity is greater in the LNVM context compared to Black-Scholes when calibrated with negative shift parameter, for example in equity markets. Models calibrated for FX markets are likely to show similar levels of option sensitivity for LNVM and Black-Scholes, at least for ATM options, while LNVM models calibrated for interest rate markets (i.e. with positive shift parameter) and likely to exhibit lower volatility sensitivity than Black-Scholes. Furthermore, the impact of volatility changes will be felt across a much wider range of strikes, depending on whether the perturbations to the volatility term structure affect the slope of the curve or not.

Of course, some of these concerns will fall away in the context of hedging, where a replicating portfolio of vanilla options can be constructed using a consistent model framework. Here, however, the primary concern becomes parameter stability, absent which the trader could easily find himself having to radically alter hedge positions even when the market itself is relatively stable. This is a significant concern for the LNVM model, which has several degrees of freedom. Taking the simple calibration exercise described on page 9 as an illustration of the problem, it can be seen from figure 23 that the objective function (shown in just two of its four dimensions) used to calibrate the model is quite flat. It is not difficult to envisage more complex calibration problems, for example, where we use a mixture of three rather than just two lognormal distributions, which could easily result in local minima being found with very different parameter estimates.

It can readily be demonstrated that a model as complex as LNVM is highly susceptible to parameter instability, even for the relatively simple calibration problem presented here. Changing just one of the observed volatilities to create an outlier at the 4.75% caplet strike produces very different parameter estimates:

\{v1\rightarrow 0.0620721, v2\rightarrow 0.179128, \alpha\rightarrow 0.121851, \lambda_1\rightarrow 0.0593295\}, compared with
\{v1\rightarrow 0.130249, v2\rightarrow 0.198467, \alpha\rightarrow 0.153773, \lambda_1\rightarrow 0.285982\} for the original data set.

The effects are shown in the chart in figure 24. In this simple case a remedy could probably be achieved with a more sophisticated objective function that enables outliers to be dealt with by means of a weighting matrix. All the same, it is not difficult to devise scenarios where a combination of very small changes in observed market data could produce very different estimates of model parameters.
Fig 23: Objective function for simple calibration problem

Fig 24: Re-calibrated volatilities and market mid-volatilities for a two-lognormal mixture model
Conclusions

1. An attractive feature of the extended LNVM model is its ability to calibrate a wide variety of symmetric and asymmetric volatility smiles and skews.

2. The model can also replicate smile dynamics to some degree, showing more pronounced smiles at shorter maturities, as observed in market volatilities. It is unlikely, however, that it can successfully replicate the very pronounced smiles at near-term maturities, which are more successfully captured by a (stochastic) jump-diffusion model.

3. LNVM is likely to be more successful in modeling the dynamics of symmetric, stable smiles in FX markets than the unstable negative skews in equity markets or positive skews in interest rate markets.

4. LNVM is a sticky-delta model, where the smile “floats” with the level of the underlying, and this results in deltas higher than Black-Scholes. Depending on whether the current regime can be characterized as driven by “greed” or “fear”, many markets display dynamical smile behavior best modeled with a sticky strike, or sticky implied tree model, respectively. In these circumstances LNVM is likely to produce (very) inflated delta values during strong up-trends and even more so in periods of market stress.

5. As far as interest rate markets are concerned, not only are skews typically very sticky, but there is the added complication that they are likely to be calibrated with positive shift parameter under LNVM. This will inflate not only deltas, but also gammas, relative to those derived from Black-Scholes, to an even greater extent than in markets where the LNVM model calibrates with a zero or negative shift parameter value. The rate of decay of time value is also positively correlated with the shift parameter.

6. Volatility sensitivity is highly dependent on the shift parameter used to calibrate the LNVM model, with which it is inversely correlated, and the impact of volatility changes will be felt across a wider range of strikes if the slope of the volatility term structure changes. Managing the changes in volatility sensitivity of an exotics portfolio modeled with LNVM is likely to prove extremely challenging, given its high degree of dependency on key model parameters and assumptions about how perturbations affect the volatility term structure. These difficulties will be exacerbated for longer-maturities and this may limit the practical applicability of the model for longer-dated securities.

7. LNVM is a complex model with many degrees of freedom, including the choice of n lognormal distributions, n volatility parameters, n-1 weighting parameters and a shift parameter. Parameter instability is likely to arise because of the flatness the objective function around the global minimum, resulting in parameter values associated with local minima. Brigo and Mercurio recommend a two-stage search procedure, using a local optimization search around the minimum identified by an initial global search. An objective function that penalizes for the number of parameters, reduces the influence of outliers and controls the degree of over-fitting would probably also help stabilize parameter estimates.

References