Lab: Testing the Purchasing Power Parity Theorem

In this lab we discover how unit root tests can be applied to test the validity of the Purchasing Power Parity Theorem.

The theorem says that a currency should depreciate by the difference between the domestic and foreign inflation rates.

A simple version of the theorem can be expressed as follows:

$$e_t = p_t - p^*_t + d_t$$

Where:

- e_t is log of dollar price of foreign exchange
- p_t is log of US price levels
- p^*_t is log of foreign price level
- d_t represents deviation from PPP in period t

According to the theory, therefore, the series $\{d_t\}$ should be stationary.

Consider now the following series, known as the real exchange rate, which is defined as:

$$r_t \equiv e_t + p^*_t - p_t$$

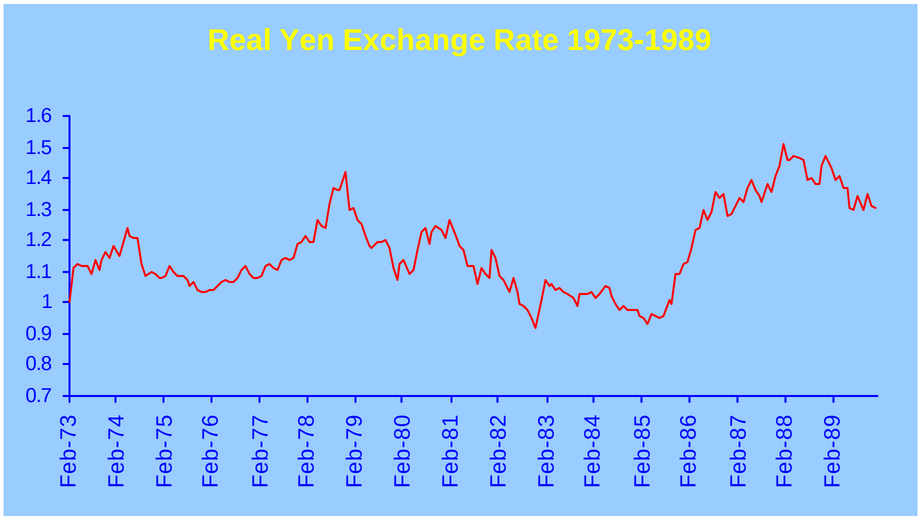
We can test whether purchasing power parity holds in the long run by determining whether or not the series $\{r_t\}$ is stationary.

To carry out the test we are going to use monthly data for the Japanese Yen, over the period from 1973 to 1989 (see chart below). The data series was constructed as:

$$r_t = Ln(S_t \times WPI^{JP}_t / WPI^{US}_t)$$

Where,

- S_t is the Yen spot rate at time t
- WPI^{JP}_t is the is Japanese wholesale price index at time t (Feb 1973 = 100)
- WPI^{US}_t is the is US wholesale price index at time t (Feb 1973 = 100)



We have therefore transformed the problem of testing the PPP theorem into a test of whether the real exchange rate series $\{r_t\}$ is stationary. We can verify this by applying the Dickey-Fuller procedure to test for unit roots in the series, using a model of the form:

$$\Delta r_t = a_0 + \gamma r_{t-1} + \varepsilon_t$$

The unit root test involves testing the null hypothesis that γ in the above equation is zero. If we are able to reject the null hypothesis (at some appropriate confidence level, say 95%) then the series is stationary and PPP holds, at least for this data series.

If we are unable to reject H_0: $\gamma = 0$ then the series has a unit root and is non-stationary, in which case we reject the purchasing power theorem as invalid in this case.

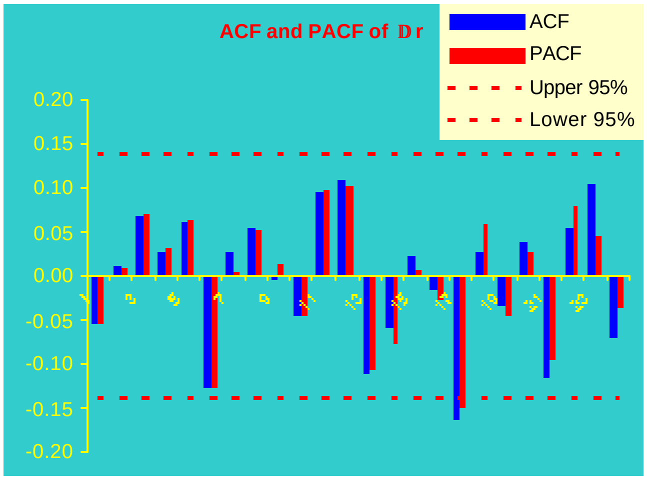
1. Examine the ACF and PACF functions for the series $\{\Delta r_t\}$. Explain why it is not possible to determine whether or not the series is stationary or not.
2. Find the maximum likelihood estimates for the parameters in the above model (including a constant term) and test the significance of the key model parameter γ using a standard t-test.
3. According to standard statistical theory, why might you be tempted to reject the hypothesis that $\gamma = 0$?
4. Perform the Dickey-Fuller test to determine whether the series has a unit root. The 95% critical value of the empirical distribution of Dickey-Fuller's T_μ statistic with 200 degrees of freedom is -2.88.

   What do you conclude about the stationary of the series and hence about the purchasing power parity theorem?

Solution: Testing the Purchasing Power Parity Theorem

1. The ACF and PACF of the series $\{\Delta r_t\}$ is shown in the chart below. While few of the coefficients appear statistically significant, there is no sign of decay at higher lags, which we would expect to see for a stationary series.

   However, it is impossible to tell simply by visual inspection whether we are dealing with a unit root non-stationary process, or simply a stationary autoregressive process with a root very close to unity. In the latter case the decay in ACF coefficients would also be imperceptible.



2. We find the maximum likelihood estimates of the regression parameters in the normal way, by using SOLVER to minimize the AIC (see below).

   We find that the ML estimates are as shown below.

	MLE	SE	t	p
a_0	0.038	0.0203	1.881	6.14%
g	-0.031	0.0173	-1.820	7.03%
m	1			
n	202			

Max Likelihood	
AIC	-291.35
BIC	-288.04
DW	2.03
R^2	1.6%
Adj. R^2	1.1%

ANOVA	DF	SS	MS	F	p
Model	1	0.0039	0.00388	3.31	7.03%
Error	200	0.2340	0.00117		
Total	201	0.2379			

Portmanteau Tests	Q(24)	p
Box-Pierce	26.83	26.32%
Ljung-Box	29.10	17.69%

3. Our results show that the critical parameter is more than 1.8 standard errors from zero, with a t-test probability of 7.03%. This means that, according to standard statistical theory, if we were to reject the hypothesis of $\gamma = 0$, we would only have a 7% chance of being wrong. By convention, we typically use the 95% rather than the 93% confidence level to make a determination that a parameter estimate is statistically significant. Nonetheless, there is apparently significant evidence *against* the hypothesis of a unit root (H: $\gamma = 0$). On this basis we may be tempted to conclude that the series is stationary and therefore that purchasing power parity holds.

4. In applying the Dickey-Fuller test, our test statistic for the parameter γ is just the normal t-statistic with value -1.820. We compare this value with the 95% percentile of the empirical distribution of the Dickey-Fuller T_μ, which is -2.88. We therefore fail to reject the null hypothesis that $\gamma = 0$. We conclude that the series is non-stationary and that purchasing power parity therefore does not hold, at least in this case.

   Note that this conclusion is apparently contradicted by our finding the previous section. This apparent dilemma commonly occurs when analyzing a series with roots close to unity. Unit root tests do not have much power in discriminating between series which have roots close to unity and actual unit roots. The dilemma is only apparent because the two null hypotheses are quite different. It is perfectly consistent to maintain a null that PPP holds and not to be able to reject a null that PPP fails!